Conical Perspective and Fractal Theory: A Comparative and Contrastive Approach

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Abstract: This paper explores a possible connection between Euclidean geometry, which lies at the basis of conical perspective, and fractal geometry, which could, in turn, generate a new system of spatial representation in art. Founded by Renaissance theorists and artists and applied exclusively to the visual arts as the only method of shaping the pictorial space for nearly five centuries, conical perspective has been increasingly questioned by modern artists. As a system of geometric relationships, conical perspective was based on the principles of Euclidean geometry. The new concepts of non-Euclidean geometry emerging in the second half of the 19th century have led to a change in the artists' perception of space, generating a quest for new ways of spatial representation. In the 1970s, Benoit Mandelbrot theorised a new type of geometry – fractal geometry – which subsequently became a second anti-Euclidean revolution that led to an unprecedented positioning of visual artists with regard to the expression of spatiality. From this point of view, fractal geometry can be seen as another system of visual representation of reality, alongside the already established ones.

Key words: conical perspective, Euclidean geometry, fractal geometry, fractals, visual representation of space.

Introduction

The role of the systems of spatial representation in visual art is that of depicting the three-dimensional material reality on a flat surface. While parallel perspective provides an objective image of the concrete reality without the involvement of an observer, conical perspective explores it from a subjective point of view.

Nowadays, conical perspective is generally accepted as being the only system of visual representation that succeeds in generating a faithful image of the material reality, according to the human visual apparatus, by means of its geometric instruments. In many cases, however, artists representing different eras and artistic styles have sought other methods to

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render space, in line with their own world view, determined by philosophical theories, scientific discoveries, and technological developments.

According to the Renaissance theory, the visual representation in conical perspective was correct and natural because it corresponded to human sight, the painted image being a mirror of the real world and the painting itself the surface of the mirror. This type of perspective seemed to be the right answer to all the questions concerning the problem of the illusionistic representation of space. Conical perspective is the result of the efforts of generations of theorists and artists who have attempted to provide a satisfactory method of rendering reality.

The success of conical perspective, which was used as a system for constructing pictorial space for almost five centuries, was also due to conjunctural factors. According to art historian René Passeron,¹ they are as follows:

- the high repute of the Renaissance painters, who developed the principles of perspective and applied them in their own creations during the 14th and 15th centuries.
- the Renaissance theory, elaborated by Brunelleschi, Alberti, and Leonardo da Vinci, which gave perspective a scientific foundation and considered it to be a *true* method of representing reality.
- the European art academies, which regarded perspective as a system whose doctrine aimed to promote the rules of truth and beauty.

Regardless of the fact that conical perspective is still considered to be the only generally accepted system of visual representation, most painters have abandoned it in this day and age. Moreover, even some Renaissance painters were far from strictly applying the principles of Alberti's perspective. The use of perspective with the aim of creating the illusion of concrete reality was paramount for those who developed it in the *Quattrocento*, but some

artists were willing to deviate from the rules because they led to unsightly distortions and unwelcome coercion of subject matter and expression when applied mechanically. (...) Modifications of this kind are applied intuitively in order to make the picture fit the intended expression or look more natural².

This is the case of artists such as Filippo Lippi, Mantegna, Gozzoli, Bellini and even Piero della Francesca.

At the end of the 19th century, perspective began to be increasingly challenged by painters, who gave up applying its principles. This change in perception was brought about by several factors.

¹ René Passeron - Opera picturală, Edit. Meridiane, București, 1982, pp. 179-180.

² Rudolf Arnheim – *Art and Visual Perception. A Psychology of the Creative Eye. The New Version*, University of California Press, Berkeley, Los Angeles, London, 2004, p. 299.

The first factor was the emergence of a new concept of space as a result of technological developments. The increase in the speed of movement determined the Futurist painters to recreate the sensation of motion and to render the evolution of the visual form in relation to the concept of time. In order to fulfil this desideratum, the Futurists needed to abandon the single viewpoint and the immobility of the observer. The same was true in the case of the Cubist painters, who adopted polycentrism with the aim of portraying novel aspects of the shape of the object and its evolution in space, by giving the observer the opportunity to simultaneously see the subject matter from several viewpoints.

Another factor that led to the decline of conical perspective in the modern era was the rediscovery of ancient and medieval art. In these creations, the modern painters identified ways of visually expressing space that fit their own vision. Thus, they discovered possibilities for configuring the space of the artwork that did not follow the principles of perspective. And what is more, they observed that although Egyptian, Byzantine or Romanesque art did not use conical perspective, the creations of these cultures were characterised by their own forms of representation, which seemed to be effective and expressive.

The idea that the principles of the conical perspective provide the only correct way of representing the world, in accordance with the human visual system, was also questioned by some theories from the early 20th century. The most important arguments brought into discussion were the curved surface of the retina³, the immobile eye, and the fact that images painted according to the rules of perspective should not be accepted as natural⁴. These theories attempted to show that the rendering of space is conventional, in the sense that every system of spatial representation must be learned and that such a system can be chosen by the artist as he sees fit, because there is no inherent connection between the visual images of the objects and the objects themselves.

In *Languages of Art*, philosopher Nelson Goodman states that: "the behaviour of light sanctions neither our usual nor any other way of rendering space; and perspective provides no absolute or independent standard of fidelity"⁵. Goodman points out that some of the conditions required by perspective (monocular view, the immobile eye, the viewpoint at an established distance) are too artificial and, thus, impossible to achieve.

Another argument that Goodman gives against conical perspective as a faithful way of depicting reality refers to the representation of parallel lines.

³ The argument is supported by Erwin Panofsky in *Perspective as Symbolic Form*, Zone Books, New York, 1991, pp. 31 – 36.

⁴ Arguments presented by Nelson Goodman in *Languages of Art*, The Bobbs-Merrill Company, Inc., Indianapolis, 1968, pp. 10 - 19.

⁵ Nelson Goodman – op. cit., p. 19.

One of the rules of perspective states that any two parallel lines in space should be drawn as converging, which confirms the statement of art historian Erwin Panofsky: "all parallels, in whatever direction they lie, have a common vanishing point"⁶. Goodman highlights the fact that this is not the case (referring to the vertical parallel lines in the frontal plane), precisely because of the conventional nature of perspective⁷. In his opinion, "pictures in perspective, like any others, have to be read; and the ability to read has to be acquired"⁸.

Philosopher Klaus Rehkämper⁹ refutes the three arguments brought against conical perspective as a faithful system of visual representation of the spatial relations between objects. Accepting, however, that these arguments are reasonable, Rehkämper proves that conical perspective accurately depicts concrete reality because the theory on which it is based explains human vision in a correct manner. In his view, images rendered in conical perspective do not belong to a language whose symbols are chosen by convention:

These pictures have as a core the natural system of linear perspective – a system that also describes correctly the way the human visual system works – and that is why representational pictures of this kind are much easier to read under normal conditions than any language is¹⁰.

From this point of view, other systems of visual representation of the spatial relationships between objects, such as the Egyptian perspective or the Byzantine reverse perspective, can be described as incorrect or primitive. However, as we have already shown, the painters who used these systems did not pursue an illusionistic representation of concrete reality. The same was true in the modern period, when painters aimed at visually representing the aspects of reality that escape direct observation, the effects of this reality on their psyche, the "reality" of dreams and of the human subconscious, or even the "imagined realities". Thus, we can refer to the relativity of the systems of spatial representation in relation to the reality that the artist wishes to translate into images.

In modern and contemporary art, the systems of visual representation of space are often "juxtaposed", as the artists are driven by the desire to convey certain meanings or to add plastic expressiveness to their works¹¹. In

⁶ Erwin Panofsky - Perspective as Symbolic Form, Zone Books, New York, 1991, p. 28.

⁷ Nelson Goodman – op. cit., p. 16.

⁸ Ibidem, p. 14.

⁹ Klaus Rehkämper – *What You See is What You Get - The Problem of Linear Perspective* in *Looking into Pictures*, edited by Heiko Hecht, Robert Schwartz, Margaret Atherton, MIT Press, 2003, pp. 184-189.

¹⁰ *Ibidem*, p. 189.

¹¹ Cătălin Soreanu, Lavinia German - From an Exhibition Gallery to a Space for Contemporary Art Projects. Aparte Gallery of UNAGE Iași în Review of Artistic Education no. 24, Artes, Iași, 2022, pp. 204-214.

many cases, terms such as "anti-perspective", "aperspectival drawing" or "non-perspectival" are used to describe those creations that do not use the principles of conical perspective in the representation of space. The painter Zamfir Dumitrescu considers that anti-perspective is omnipresent in the plastic creation of the 20th century, and, by extension, of all painters¹².

This opinion demonstrates once again the prestige of perspective as an accurate system of visual representation of the spatial relations between objects in the concrete world. The "aperspectival" world can be built on the foundations of the perspectival world in order to surpass it. The "aperspectival" cubist image is, in fact, a juxtaposition of the various hypostases of the process of visual contemplation of the object, a sum of perspectival images.

The new scientific and philosophical theories of the second half of the 19th century and the first part of the 20th century radically changed man's perception of the universe and redefined the notion of space, also having a strong impact on the evolution of visual arts.¹³. These theories demonstrated the need for knowledge systems not to be based on intuitive perceptions of space and time. In this context, the theories of Riemann, Lobacevski and Bolyai formulated new methods to define and visualise non-Euclidean spacetime concepts.

The systems of visual representation of space can also be analysed in terms of their relationship with the underlying geometric theories. As previously mentioned, conical perspective is based on the principles of Euclidean geometry. Byzantine reverse perspective can also be explained in relation to Euclidean geometry, just as Egyptian perspective is rooted in the principles of descriptive geometry. The methods of spatial representation used in modern painting are influenced by non-Euclidean geometries.

In the second half of the 20th century, research in physics and mathematics led to the development of new methods for investigating the world of shapes, such as René Thom's catastrophe theory¹⁴, Ilya Prigogine's dissipative structures¹⁵, David Ruelle's theory of chaos and strange attractors¹⁶, Hermann Haken's synergetics¹⁷, or Benoît Mandelbrot's theory

¹² Zamfir Dumitrescu – Ars perspectivae, Edit. Nemira, București, 2002, p.73.

¹³ Mihai Vereștiuc - Object And Objecthood In Post-Minimal Sculpture in Review

of Artistic Education, no. 24, Artes, Iași, 2022, pp. 194-203.

¹⁴ René Frédéric Thom (1923-2002), French mathematician who became an important figure within the international academic comunity due to his catastrophe theory.

¹⁵ Ilya Prigogine (1917-2003), Belgian physicist and chemist of Russian descent, known for having defined the dissipative structures and their role in thermodynamic systems, a discovery that won him the Nobel Prize in 1977.

¹⁶ David Pierre Ruelle (born 1935), Belgian-French mathematician and physicist.

¹⁷ Hermann Haken (born 1927), German physicist, founder of *synergetics*, a field of science about the interaction of the component parts of a system that tends towards self-organization.

of fractals¹⁸. These research directions, known as morphological theories, caused a radical break with the established orientations of the "classical" sciences.

The morphological theories are a (phenomenological) expression of appearances. They represent a reconstruction of the universe formed by the objects of our perception¹⁹, which bears a resemblance to the quests of the numerous styles of visual art. This allows us to identify a relationship between the morphological theories and conical perspective, understood here as a science of the human vision.

Euclidean geometry, which originally used deductive methods to study flat surfaces and rigid three-dimensional objects, was an abstract, autonomous universe with no clear connections to concrete reality. Rather, it described a universe of absolute, ideal values. In this sense, the Renaissance artists' desire to construct an ideal world using the principles of conical perspective was not accidental.

The shortcoming of this system of representation is that it is based on a number of simplifying theories. The system only describes methods of spatial construction of simple geometric shapes and bodies: polygons, polyhedra, circles, spheres, etc. In terms of representing and understanding the appearance and structure of complex objects created by nature, conical perspective is limited. The same is true for classical geometry when one wishes to represent natural shapes and structures on a flat surface.

These arguments are presented by Benoît Mandelbrot when he develops fractal theory. His statement is perfectly justified: "

Why is geometry often described as 'cold' and 'dry'? One reason lies in its inability to describe the shape of a cloud, a mountain, a coastline, or a tree. Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line.²⁰

Mandelbrot points out the discrepancies between traditional geometry and nature, stating that a large part of natural forms cannot be adequately represented using notions of Euclidean geometry. He conceives and develops a new geometry of nature – fractal geometry – which he tries to implement in various fields. Mandelbrot's theory brings together the studies of mathematicians such as Waclaw Sierpinski, David Hilbert, Georg Cantor and Helge von Koch, who, between 1875 and 1925 (a period of crisis in mathematics), came across bizarre shapes that were in contradiction with

¹⁸ Benoît Mandelbrot (1924-2010), mathematician with dual citizenship - French and American (of Polish origin), considered to be the father of fractal geometry and one of the visionary scientists of the 20th century.

¹⁹ Alain Boutot – Inventarea formelor, Edit. Nemira, București, 1996, pp. 180 – 181.

²⁰ Benoît Mandelbrot – *The Fractal Geometry of Nature*, W.H. Freeman and Company, New York, 1983, p. 1.

their concepts of space, surface, distance and dimension²¹. These shapes, which defied some of the highly treasured beliefs of the mathematicians who studied them, are considered the precursors of fractals.

Fractal geometry is viewed as a second anti-Euclidean revolution, much more powerful than the first one, formulated by the non-Euclidean mathematicians of the 19th century. The object of study of this type of geometry is represented by those categories of natural forms forgotten by classical geometry, forms which are characterised by an intrinsic complexity and a fundamental irregularity that manifest themselves at all scales of observation. Fractal theory does not aim to investigate the genesis of shapes; instead, it develops a new formula for reading existing shapes. It attempts to explain phenomena such as the hierarchical structure of the universe or the irregular spread of matter, problems which had been studied frequently but had not been satisfactorily answered. Its goal does not consist in presenting a theoretical explanation of these problems; it rather attempts to simply describe them, to imitate reality by means of purely geometric tools.

With the help of fractal dimensions, a whole universe of shapes, which escapes Euclidean geometry, can be measured. As mathematician and computer scientist Dick Oliver argues, for the first time since Descartes, a completely new tool for measuring space has been created²². In this way, fractal geometry emerges as a different type of geometry and as a new way of understanding nature. Since the 1970s, many natural structures have been regarded as being fractal, and fractals have acquired the impressive title of "the fingerprint of God"²³.

The term *fractal* is a neologism coined by Mandelbrot in 1975. Its etymology²⁴ comes from the Latin word *fractus*, derived from the verb *frangere*, which means to break, to shatter, to crush into irregular fragments. The fractal is defined by the French mathematician as a geometrical structure or a concrete object combining the following characteristics:²⁵

- the parts, like the whole, have the same shape or structure, even if they have different scales.
- regardless of scale, the shape of a fractal is highly irregular, interrupted or fragmented.
- a fractal has "distinctive elements" that can be identified at any scale.

²¹ Dick Oliver - Fractali, Edit. Teora, București, 1996, p. 19.

²² Dick Oliver - Fractali, Edit. Teora, București, 1996, p. 45.

²³ Richard Taylor – *Fractal Expressionism* – *Where Art Meets Science* în J. Casti, A. Karlqvist – *Art and Complexity*, Elsevier Science, Amsterdam, 2003, p. 119.

²⁴ Benoît Mandelbrot – *The Fractal Geometry of Nature*, W.H. Freeman and Company, New York, 1983, p. 4.

²⁵ Idem – Obiecte fractale, Edit. Nemira, București, 1998, p. 72.

More specifically, a fractal is a geometric pattern that is self-similar across different scales. Its repetition produces irregular shapes or surfaces that cannot be represented by Euclidean geometry.

Artists have shown a particular interest in fractal shapes, especially after these structures became widely known. As artistic interest has grown, a new form of digital art has emerged, quickly gaining popularity both within and outside the artistic (and scientific) communities. Mathematicians Marc Frantz and Annalisa Cranell have identified a number of striking similarities between images created by artists and computer-generated pictures²⁶. Frantz and Cranell compare three details of woodblock prints by Japanese artists Ando Hiroshige, Katsushika Hokusai and Ikkasai Yoshitoshi by pairing them with three computer-generated fractal shapes. It is interesting to note that, although these pairs of images look similar, the woodblock prints predate the computer-generated images by more than a hundred years. The example shows that artists identified these fractal forms in nature and exploited their expressive potential before the advent of Mandelbrot's theory. It should also be pointed out that these images are forms of Japanese art, which resorted to the conical perspective of the Renaissance for a brief period of time. From this point of view, fractal art can be seen as another system of visual representation of reality, alongside the already established ones.

Furthermore, physicist Richard Taylor²⁷ has conducted research on Jackson Pollock's paintings, highlighting the fractal aspect of the shapes obtained by the American artist. Since Pollock's paintings are often described as organic in character, while analysing them, Taylor applied the same techniques used to study natural fractal structures. Using computer programs²⁸, he demonstrated the striking similarity between Pollock's drip paintings, certain natural structures and computer-generated artificial fractals. At the same time, Taylor suggested replacing the term Abstract Expressionism with *Fractal Expressionism* in reference to Pollock's work. In his view, *Fractal Expressionism* indicates the ability to generate and manipulate fractal models directly²⁹.

In line with this idea, we could go even further and propose the term *fractal perspective* to describe the innovative method of spatial representation

²⁶ Marc Frantz, Annalisa Crannel – *Viewpoints: mathematical perspective and fractal geometry in art*, Princeton University Press, New Jersey, 2011, p. 142.

²⁷ Richard P. Taylor – *Fractal Expressionism* – *Where Art Meets Science* in J. Casti, A.

Karlqvist - Art and Complexity, Elsevier Science, Amsterdam, 2003, pp. 117-144.

²⁸ Cătălin Soreanu – New Media Art: Aligning Artistic Creativity and Technological Media, în Review of Artistic Education, no. 22, Artes, Iași, 2021, pp. 206-216.

²⁹ R. P. Taylor, et al. – Authenticating Pollock Paintings Using Fractal Geometry, Elsevier, Pattern Recognition Letters, no. 28, 2007, pp. 695-702. Source:

https://www.academia.edu/76239188/Authenticating_Pollock_paintings_using_fractal_geometry

that contemporary artists³⁰ have been increasingly using and that heavily relies on fractal geometry.

Thus, if abstract painting means abandoning the conical perspective of the Renaissance as a system of spatial representation, fractal geometry is one of the solutions that painters could resort to when configuring the plastic space of the painting. Euclidean geometry and conical perspective (the latter being grounded on the principles of the former) are only necessary in figurative art, especially for rendering simply shaped elements. By contrast, fractal geometry can be used both for representing complex and very diverse forms of nature and for configuring abstract spaces. A fractal form of expression does not design linear systems or configure ordered sets of points. The artist removes the conventional relationships between him and the visual field – relationships that are based on perspective and the usual notions of the Euclidean model. He builds an irregular space in which each element contributes to the creation of a broken, discontinuous and asymmetrical fractal form.

Conclusions

Starting from flat fractal structures, mathematicians and artists have been able to model three-dimensional shapes and create virtual spaces resembling the real one. The advent of the computer has brought conical perspective and fractal geometry together, in a joint effort to study the universe. In both art and science, conical perspective and fractal geometry are committed to the process of investigating concrete reality. The two types of geometric approaches, although different, complement each other. In visual art, fractal geometry can be seen as an additional system of visual representation of reality, alongside the already established ones, such as parallel perspective and conical perspective. Given the increasing number of artists who have been using fractal geometry in their exploration and visual expression of the material or imagined realities, we could use the term *fractal perspective* to describe this method of spatial representation that has been emerging in art during the last decades.

³⁰ Cătălin Soreanu, et al., Galeria Aparte. Index 2005-2020, Iași, Galeria Aparte, UNAGE Iași, 2021.

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